Student Number:									

Surname:																
First Name(s):																

Department of Mathematics and Applied Mathematics

MAM1000W

Test 2 **3 May 2018**

Time : 18h00 – 20h00 **Full marks: 80**

- This question paper consists of 16 pages (including this one). Pages are printed on both sides
- Answer all questions in the spaces provided on this question paper.
- Use your *UCT Examination Answer Book* for rough work. The work in your *UCT Examination Answer Book* will not be marked.
- Calculators are not allowed. When your answer contains constants such as π, e or $\sqrt{2}$, leave them in that form. Don't simplify your answers.
- Be careful to provide answers that we can read and make sense of at all times. Work that is poorly presented will be penalized.

DO NOT WRITE BELOW THIS LINE

										Subtotals
						A		correct	Incorrect	
							4 times		minus	/24
1						B			B1	B2
									/12	/4 /16
C	C1	C2	C3	C4	C5	C6	C7	C8		
	/4	/4	/4	/4	/5	/8	/5	/6		/40
									Total	/80
									Check 1	initials
									Check 2	initials

Student Number:									

SECTION A: MULTIPLE CHOICE QUESTIONS [24 marks]

Marking: Correct answer = 4, no answer = 0, wrong answer = -1

Question:	A1	A2	A3	A4	A5	A6
Answer:						

Question A1. [**4 points**] The equation for the tangent to the curve $y = x^2 + \sin x$ at the point where $x = \dfrac{\pi}{2}$ is

A: $y = 1 - \dfrac{\pi^2}{4} + \pi x$ B: $y = (1 - \dfrac{\pi^2}{4})x + \pi$ C: $y = 1 + \dfrac{\pi^2}{4} + \pi x$ D: $y = (2 + \dfrac{\pi^2}{4})x + \pi$

E: $y = (2 + \dfrac{\pi^2}{4})x + \pi$

Question A2. [**4 points**] Suppose f and g are differentiable functions for which you know the following:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	4	-1	3
2	1	-3	6	2

.

Then

$$\frac{d}{dx} g\left(\sqrt{f(x)}\right)\Big|_{x=2}$$

is equal to

A: $\dfrac{9}{2}$ B: $\dfrac{3}{2}$ C: $-\dfrac{3}{2}$ D: $-\dfrac{9}{2}$ E: -9

Question A3. [**4 points**] The value of $\lim_{x\to\infty}(\sqrt{x^2 + ax} - \sqrt{x^2 + bx})$ is

A: $a - b$ B: $\dfrac{a-b}{2}$ C: $\dfrac{a+b}{2}$ D: $a + b$ E: Undefined

Question A4. [**4 points**] $\tan(\arcsin x)$ is equal to is

A: $\dfrac{x}{\sqrt{1+x^2}}$ B: $\dfrac{x}{\sqrt{1-x^2}}$ A: $\dfrac{1}{\sqrt{1+x^2}}$ B: $\dfrac{1}{\sqrt{1-x^2}}$ E: $\dfrac{1}{\sqrt{x^2-1}}$

Student Number:									

Question A5. [4 points] For the function $f(x) = \dfrac{1}{x+4}$, which of the following statements are true:

- S1: $\lim_{x\to -4^+} f(x)$ and $\lim_{x\to -4^+} f(x)$ are real numbers but are not equal.
- S2: $\lim_{x\to -4} f(x)$ does not exist.
- S3: $f(-4)$ and $\lim_{x\to -4} f(x)$ exist but are not equal.
- S4: $f(-4)$ is undefined.

A: S1, S2 and S4 are true

B: S1, S3 are S4 are true

C: S1 and S4 are true

D: S2 and S3 are true

E: S2 and S4 are true

Question A6. [4 points] Let $f(x) = \tan(x)$, $g(x) = |x-1|$ and $h(x) = \dfrac{x^2+3x}{2+4x}$. Which of them are differentiable everywhere they are defined?

A: f and g

B: f and h

C: g and h

D: f

E: None is differentiable everywhere it is defined.

Student Number:									

SECTION B: SHORT ANSWER QUESTIONS [16 marks]

Write the answers to the questions in the blocks provided. Do not show your working.

Question B1. [12 points] Calculate the following derivatives:

a. $\dfrac{d}{dx}\left(\dfrac{\tan x}{x^3}\right)$ (3)

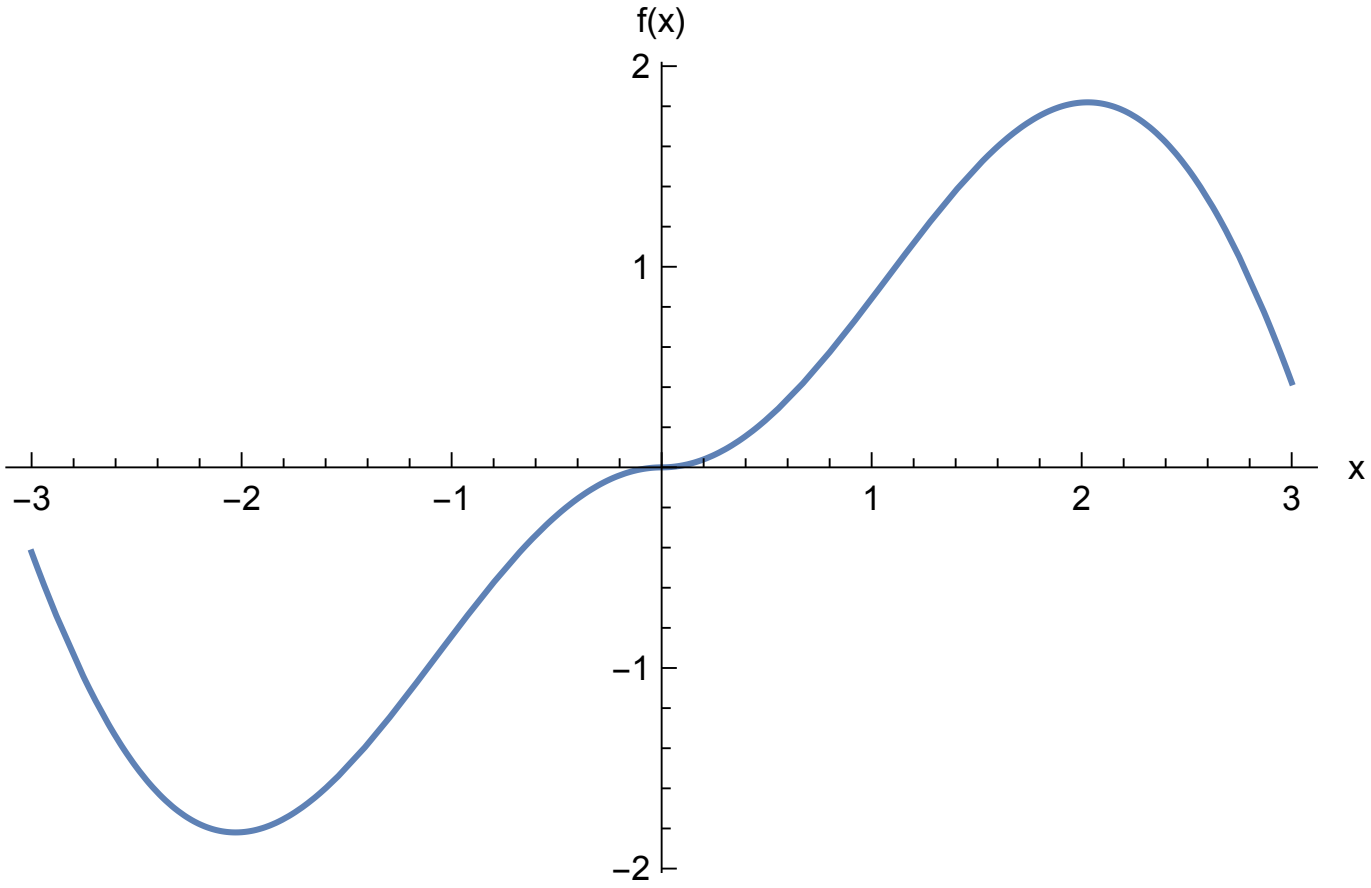
b. $\dfrac{d}{dx} x \sec x$ (3)

c. $\dfrac{d}{dx}\sqrt[3]{x}(x^2-1)$ (3)

d. $\dfrac{d}{dy}\dfrac{\sin y \cos y}{y^2}$ (3)

Student Number:									

Question B2. [4 points] On the following graph of $y = f(x)$, sketch the graph of $y = f'(x)$.



Student Number:									

SECTION C: LONG ANSWER QUESTIONS [40 marks]

Write the answers to the questions in the spaces provided.

Show all working!

Question C1. [4 points] If a function f is given by $f(x) = \dfrac{3}{x-4}$, find any points of intersection of the graph of f with the graph of the inverse function f^{-1}.

Student Number:									

Question C2. **[4 points]** Evaluate the limit

$$\lim_{x \to 0} \frac{\sqrt{x^2 + 16} - 4}{x^2} .$$

(you may not use L'Hospital's rule)

Student Number:									

Question C3. **[4 points]** Prove that there is a real number x such that $3^x + \sin(x) = 4$.

Student Number:									

Question C4. [4 points] Use the squeeze theorem and the limit laws to find the value of

$$\lim_{x \to 0} \left(x^2 e^{\sin\left(\frac{1}{x}\right)} + 4\right).$$

Student Number:									

Question C5. **[5 points]** Prove by contradiction that $\sqrt[3]{2}$ is irrational.

Student Number:									

Question C6. [8 points] Define

$$f(x) \;=\; \begin{cases} x^3 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

a. Show that $f(x)$ is continuous at $x = 0$. (4)

Student Number:									

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b. Use first principles to decide whether $f(x)$ is differentiable at $x = 0$. (4)

Student Number:									

Question C7. [5 points] Use proof by induction to show that for all integers $n \geq 2$

$$\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right) ... \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$$

Student Number:									

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Student Number:									

Question C8. [6 points] Prove, from first principles, that if $f(x)$ is a differentiable function, then for $f(x) \neq 0$, so is the function g defined by $g(x) = \frac{x}{f(x)}$, and that $g'(x) = \dfrac{1}{f(x)} - \dfrac{xf'(x)}{f(x)^2}$.

In answering this question, you may not use, quote or prove the general rule for the derivative of a product or quotient of functions or the composition of functions.

Student Number:									

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